High School Mathematics Standards

Number and Quantity

1. **The Real Number System N-RN**
   a. Extend the properties of exponents to rational exponents
      i. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{\left(\frac{1}{3}\right)}$ *to be the cube root of 5 because we want* $\left(5^{\frac{1}{3}}\right)^3 = 5^{\left(\frac{1}{3}\right)(3)} to\ hold, so \left(5^{\frac{1}{3}}\right)^3\ must\ equal\ 5.$
      ii. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
   b. Use properties of rational and irrational numbers.
      i. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
2. **Quantities N-Q**
   a. Reason quantitatively and use units to solve problems.
      i. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
      ii. Define appropriate quantities for the purpose of descriptive modeling.
      iii. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
3. **The Complex Number System N-CN**
   a. Perform arithmetic operations with complex numbers.
      i. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.
      ii. Use the relation $i^2 = -1$and the communicative, associative, and distributive properties to add, subtract, and multiply complex numbers.
      iii. $(+)$ Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
   b. Represent complex numbers and their operations on the complex plane.
      i. $(+)$ Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).
      ii. $(+)$ Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $\left(-1 + \sqrt{3}\,i\right)^3 = 8$ *because* $\left(-1 + \sqrt{3}\right)$ *has modulus 2 and argument 120°.*
      iii. $(+)$ Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
   c. Use complex numbers in polynomial identities and equations.
      i. Solve quadratic equations with real coefficients that have complex solutions.

ii. $(+)$ Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
iii. $(+)$ Know the Fundamental Theorem of Algebra; show that it is true for
iv. quadratic polynomials.

4. **Vector and Matrix Quantities N-VM**
   a. Represent and model with vector quantities.
      i. $(+)$ Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
      ii. $(+)$ Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
      iii. $(+)$ Solve problems involving velocity and other quantities that can be represented by vectors.
   b. Perform operations on vectors.
      i. $(+)$ Add and subtract vectors.
         1. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
         2. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
         3. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of *w*, with the same magnitude as *w* and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
      ii. $(+)$ Multiply a vector by a scalar.
         1. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$.
      iii. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of *cv* knowing that when $|c|v \neq 0$, the direction of *cv* is either along *v* (for $c > 0$) or against *v* (for $c < 0$).
   c. Perform operations on matrices and use matrices in applications.
      i. $(+)$ Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
      ii. $(+)$ Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
      iii. $(+)$ Add, subtract, and multiply matrices of appropriate dimensions.
      iv. $(+)$ Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
      v. $(+)$ Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
      vi. $(+)$ Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

vii. $(+)$ Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

5. Seeing Structure in Expressions A-SSE

a. Interpret the structure of expressions
   i. Interpret parts of an expression, such as terms, factors, and coefficients.
   ii. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
   iii. Use structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as$(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

b. Write expressions in equivalent forms to solve problems
   i. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
      1. Factor a quadratic expression to real the zeros of the function it defines.
      2. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
      3. Use the properties of exponents to r=transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $\left(1.15^{\frac{1}{2}}\right)^{12t}$ *to reveal the approximate equivalent monthly interest rate of the annual rate is 15%.*
   ii. Derive the formula for the sum of a finite geometric series)(when the common ratio is not 1), and use the formula to solve problems. *For example calculate mortgage payments.*

6. Arithmetic with Polynomials and Rational Expressions A-APR

c. Perform arithmetic operations on polynomials
   i. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

d. Understand the relationship between zeros and factors of
   i. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number *a*, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
   ii. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

e. Use polynomial identities to solve problems
   i. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*
   ii. $(+)$ Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.

f. Rewrite rational expressions

i. Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $\frac{q(x)+r(x)}{b(x)}$, where $a(x), b(x), q(x), and\ r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection long division of, for he more complicated examples, a computer algebra system.

ii. $(+)$ Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

7. Creating Equations A-CED

g. Create equations that describe numbers or relationships

i. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

ii. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

iii. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

iv. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

8. Reasoning with Equations and Inequalities A-REI

h. Understand solving equations as a process of reasoning and explain the reasoning

i. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

ii. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

i. Solve equations and inequalities in one variable

i. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

ii. Solve quadratic equations in one variable.

1. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
2. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

j. Solve systems of equations

i. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

ii. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
iii. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y =- 3x$ *and the circle* $x^2 + y^2 = 3$.
iv. $(+)$ Represent a system of linear equations as a single matrix equation in a vector variable.
v. $(+)$ Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

k. Represent and solve equations and inequalities graphically
   i. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
   ii. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
   iii. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions

1. Interpreting functions F-IF

a. Understand the concept of a function and use function notation.
   i. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, the $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.
   ii. Use function notation, evaluate functions for inputs in their domains, and interpret statement that use function notation in terms of a context.
   iii. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1)\ for\ n \geq 1$.

b. Interpret functions that arise in applications in terms of the context
   i. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
   ii. Relate the domain of a function to its graph and, where applicable, to the quantities relationship it o its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* $h(n)$ *gives the number of person-hours it*

takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

iii. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

c. Analyze functions using different representations

i. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

1. Graph linear and quadratic functions and show intercepts, maxima, and minima.
2. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
3. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
4. $(+)$ Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
5. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

ii. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

1. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
2. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$*, and classify them as representing exponential growth or decay.*
3. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

2. Building Functions F-BF

d. Build a function that models a relationship between tow quantities

i. Write a function that describes a relationship between two quantities.

1. Determine an explicit expression, a recursive process, or steps for calculation from a context.
2. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
3. $(+)$ Compose functions. *For example, if* $T(y)$ *is the temperature in the atmosphere as a function of height, and* $h(t)$ *is the height of a weather balloon as a function of time, then* $T(h(t))$ *is the temperature at the location of the weather balloon as a function of time.*

ii. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

e. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

3. Build new functions from existing functions

f. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

g. Find inverse functions.

   i. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$.

   ii. $(+)$ Verify by composition that one function is the inverse of another.

   iii. $(+)$ Read values of an inverse function from a graph or a table, given that the function has an inverse.

   iv. $(+)$ Produce an invertible function from a non-invertible function by restricting the domain.

h. $(+)$ Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

4. Linear, Quadratic, and Exponential Models F-LE

i. Construct and compare linear quadratic and exponential models and solve problems

   i. Distinguish between situations that can be modeled with linear functions and with exponential functions.

      1. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
      2. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
      3. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
      4. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
      5. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
      6. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is $2, 10$, or e; evaluate the logarithm using technology

j. Interpret expressions for functions in terms of the situation they model

   i. Interpret the parameters in a linear or exponential function in terms of a context.

5. Trigonometric Functions F-TF

k. Extend the domain of trigonometric functions using the unit circle

   i. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

   ii. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

iii. $(+)$ Use special triangles to determine geometrically the values of *sine, cosine*, *tangent* for $\frac{\pi}{3}, \frac{\pi}{4}$ and $\frac{\pi}{6}$, and use the unit circle to express the values of *sine, cosine*, and *tangent* for $\pi - x, \pi + x$, and $2\pi - x$ in terms of their values for *x*, where *x* is any real number.
iv. $(+)$ Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

l. Model periodic phenomena with trigonometric functions
   i. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
   ii. $(+)$ Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
   iii. $(+)$ Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

m. Prove and apply trigonometric identities
   i. Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta), cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.
   ii. $(+)$ Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.

- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena. Modeling Standards Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

Statistics and Probability

1. **Interpreting Categorical and Quantitative Data S-ID**
   a. Summarize, represent, and interpret data on a single count or measurement variable
      i. Represent data with plots on the real number line (dot plots, histograms, and box plots).
      ii. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
      iii. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
      iv. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for

which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

b. Summarize, represent, and interpret data on two categorical and quantitative variables
    i. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
    ii. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
        1. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
        2. Informally assess the fit of a function by plotting and analyzing residuals.
        3. Fit a linear function for a scatter plot that suggests a linear association.

c. Interpret linear models
    i. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
    ii. Compute (using technology) and interpret the correlation coefficient of a linear fit.
    iii. Distinguish between correlation and causation.

2. Making Inferences and Justifying Conclusions S-IC

d. Understand and evaluate random process and underlying statistical experiments
    i. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
    ii. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

e. Make inferences and justify conclusions from sample surveys, experiments, and observational studies
    i. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
    ii. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
    iii. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
    iv. Evaluate reports based on data

3. Conditional Probability and the Rules of Probability S-CP

f. Understand independence and conditional probability and use them to interpret data
    i. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
    ii. Understand that two events *A* and *B* are independent if the probability of *A and B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
    iii. Understand the conditional probability of *A given B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A given B* is

the same as the probability of *A*, and the conditional probability of *B given A* is the same as the probability of *B.*

   iv. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

   v. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

g. Use the rules of probability to compute probabilities of compound events in a uniform probability model

   i. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

   ii. Apply the Addition Rule, $P(A\ or\ B) = P(A) + P(B) – P(A\ and\ B)$, and interpret the answer in terms of the model.

   iii. $(+)$ Apply the general Multiplication Rule in a uniform probability model, $P(A\ and\ B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

   iv. $(+)$ Use permutations and combinations to compute probabilities of compound events and solve problems.

4. Using Probability to Make Decisions S-MD

h. Calculate expected values and use them to solve problems

   i. $(+)$ Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

   ii. $(+)$ Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

   iii. $(+)$ Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

   iv. $(+)$ Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

i. Use probability to evaluate outcomes of decisions

   i. $(+)$ Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

1. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
2. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
3. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
4. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).